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PERFORMANCE ANALYSIS OF BATCH ARRIVAL QUEUE UNDER FUZZY ENVIRONMENT

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Abstract
The main aim of this paper is analyzing various performance measure interms of crisp values for the total expected cost of fuzzy batch arrival queuing model where the arrival rate, service rate and batch size, service cost and holding costs are all fuzzy numbers and also they are Triangular fuzzy numbers, Trapezoidal fuzzy numbers. Here we convert the fuzzy inter arrival rate, service rate, batch size service cost are all into crisp values for using ranking function method.

Keywords: Fuzzy Sets, Batch Arrival Queue, Fuzzy Ranking Membership Function.

1. Introduction

A queue is formed at any time when the demand for a service exceeds the capacity to provide that service. Queuing models of computer and telecommunication systems, manufacturing / production systems and inventory control are discussed by many researchers. In this paper, a study of FM[K]/FM/1 type Queuing models many types of batch arrival . Queuing models has been investigated so far. Then [s,t] developed FM/FM/1 and FM/FM[K]/1 Fuzzy systems. Let C.H Hung H.L., Ke. J.C [78] developed parametric programming Approach for Batch arrival Queues with vaccation policies and Suzy parameters and also discussed. On a Batch Arrival Queue with setup and uncertain parameters patterns. Recently Rue and Rostine [10] death with the control policy on Batch arrivals and Lilly Robert [11] developed the profit analysis of fuzzy M/Ek/1 Queuing system by using non - linear programming technique. Thus in this paper rare develop a method which is able to provide TEC for bulk arrival queues with Fuzzified exponential arrival rate, service time, batch size, service cost and holding cost. Abbasbandy and Hajjari [24] introduced a new approach for ranking of Trapezoidal Fuzzy numbers based on left and right spread at some α – levels of Trapezoidal Fuzzy number.

2. Definition : Triangular Fuzzy Number

A Triangular Fuzzy number X$(\overline{x})$ can be represented by three parameters by

$$\mu(\overline{x}) = \begin{cases} \frac{x-x_1}{x_2-x_1}, & 0 < x < x_1 \\ 1, & x = x_2 \\ \frac{x-x_3}{x_2-x_3}, & x_2 \leq x \leq x_3 \\ 0, & \text{Otherwise.} \end{cases}$$

3. Definition : Trapezoidal Fuzzy number

A Trapezoidal Fuzzy number X$(\overline{x})$ can be represented by four parameters (x_1, x_2, x_3, x_4) with membership function $\mu_{\overline{x}}(x)$ is given by

$$\mu_{\overline{x}}(x) = \begin{cases} \frac{x-x_1}{x_2-x_1}, & x_1 \leq x \leq x_3 \\ 1, & x_2 \leq x \leq x_3 \\ \frac{x-x_4}{x_3-x_4}, & x_3 \leq x \leq x_4 \\ 0, & \text{Otherwise.} \end{cases}$$

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4. Model Description

Cost Measures of Fuzzy Batch Arrival Queue

Consider a Fuzzy batch Arrival Queuing model for finding Total Expect Cost (TEC). It is assumed that the customers arrive at a batch arrival as a poisson process with Fuzzy rate $\overline{\lambda}$, the service torises as an exponential distributions with Fuzzy rate $\overline{\mu}$, the expected batch size with Fuzzy rate $\overline{k}$, the service cost with Fuzzy rate $\overline{C_s}$ and the holding cost with Fuzzy rate $\overline{C_h}$ are known approximately and can be represented by convox Fuzzy sets.

Let $\phi_{\overline{\lambda}}(u),\ \phi_{\overline{\mu}}(v),\ \phi_{\overline{k}}(u),\ \phi_{\overline{C_s}}(m),\ and\ \phi_{\overline{C_h}}(n),$ denotes the membership functions of $\overline{\lambda}, \overline{\mu}, \overline{k}, \overline{C_s}, and\ \overline{C_h}$ respectively. Then we have the following sets,

$$\overline{\lambda} = \{(u, \phi_{\overline{\lambda}}(u))/u\epsilon U\}$$
$$\overline{\mu} = \{(u, \phi_{\overline{\mu}}(v))/v\epsilon V\}$$
$$\overline{K} = \{(w, \phi_{\overline{k}}(w))/w\epsilon W\}$$
$$\overline{C_s} = \{(m, \phi_{\overline{C_s}}(m))/m\epsilon M\}$$
$$\overline{C_h} = \{(n, \phi_{\overline{C_h}}(n))/n\epsilon N\}$$

Where U, V, W, K, M, N are the crisp Universal sets of the arrival, service, batch size, service cost, and holding cost rates respectively.

Let F(u, v, w, k, m, n) denote the system characteristic of the total expected cost.

(i.e) The total expected cost (TEC) is

A = F(u, v, w, k, m, n) = m + wn $\left[\frac{E(w)^2 + E(w)}{2\{v-u\ E(w)\}}\right]$ ----------------- (1)

5. Algorithm : Ranking Function method

(i) Let a convex Trapezoidal Fuzzy number $\overline{X}(x)$. $\overline{X}(x) = \overline{X}(x_1, x_2, x_3, x_4; w)$. Then the Ranking Index is defined by,

$$R(\overline{x}) = \int_0^w \frac{[L^{-1}(x) + R^{-1}(x)]}{2}\, dx$$

Where, $L^{-1}(x) = x_1 + \frac{(x_2 - x_1)}{W} x$ and $R^{-1}(x) = x_3 + \frac{(x_4 - x_3)}{W} x$

$$R(\overline{x}) = \frac{1}{4}\,[W(x_1 + x_2 + x_3 + x_4)]$$ --------------------- (2)

(ii) Let a convex Triangular Fuzzy number

$$\overline{X}(x) = \overline{X}(x_1, x_2, x_3; w)$$

Then the Ranking Index is given by

$$R(\overline{x}) = \int_0^w \frac{1}{2} L^{-1}(x) + R^{-1}(x)\, dx$$

Where $L^{-1}(x) = x_1 + \frac{1}{W}\,[x_3 - x_1]\, x$ and $R^{-1}(x) = x_2 + \frac{1}{W}\,[x_3 - x_2]\, x$

$$R(\overline{x}) = \frac{1}{4}\,[W(x_1 + x_2 + x_3)]$$ --------------------- (3)

6. Numerical Example

Consider a TV service centre system in which TV sets are arrived by batches a according to a poisson process. In that situation all TV sets expect the service processing TV set are needed to hold in service. So the holding cost exits owner of the centre wishes to evaluate how much the total expected cost for whole service. It is evident that this system follows $FM^{[k]}/FM/1$ and the total expected cost of the system can be derived by the proposed approach.

6.1. For Trapezoidal Fuzzy Number

(i) When W = 0.5 that is generalized fuzzy number will give moderate value1.

Suppose the arrival rate, service rate, service rate, batch size and service cost and holding cost rates are Trapezoidal Fuzzy number represented by,

$$\overline{\lambda} = [2,3,4,5;0.5], \overline{\mu} = [3,4,5,6;0.5]$$
$$\overline{K} = [6,7,8,9;0.5] \qquad \overline{C_s} = [2000,\ 3000,\ 4000,\ 5000;\ 0.5]$$
$$\overline{C_k} = [40,\ 50,\ 60,\ 70;\ 0.5]$$

According to (2), the Ranking Index of $\overline{\lambda}$ is

$$R(\overline{\lambda}) = R(2,3,4,5;0.5)$$
$$= \frac{1}{4}[0.5\ (2+3+4+5)]$$
$$= 1.75 = u.$$

Similarly, we get other parameters as follows,

$$R(\overline{\mu}) = R(3,4,5,6;0.5)$$
$$= \frac{1}{4}[0.5\ (3+4+5+6)]$$
$$= 2.25 = v.$$
$$R(\overline{K}) = R(6,7,8,9;0.5)$$
$$= \frac{1}{4}[0.5\ (6+7+8+9)]$$
$$= 3.75 = w.$$
$$R(\overline{C_s}) = R(2000,\ 3000,\ 4000,\ 5000;\ 0.5)$$
$$= \frac{1}{4}[0.5\ (2000+3000+4000+5000)]$$
$$= 1750 = m.$$
$$R(\overline{C_h}) = R(40,50,60,70;\ 0.5)$$
$$= \frac{1}{4}[0.5\ (40+50+60+70)]$$
$$= 27.5 = n.$$

Using the above values in (1) we get,

The Total Expected Cost is,

$$TEC = m + wn\left[\frac{E(w)^2 + E(w)}{2\{v-u\,E(w)\}}\right]$$
$$= 1750 + (3.75)(27.5)\left[\frac{(3.75)^2 + 3.75}{2\{2.25-(1.75)\,(3.75)\}}\right]$$
$$= 1537.$$

6.2. When w = 1, that is normalized Fuzzy number will gives the optimistic value.

Suppose the arrival rate, service rate, batch size service cost rate and holding cost rate are Trapezoidal Fuzzy number represented by,

$\overline{\lambda} = [2, 3, 4, 5; 1]$, $\overline{\mu} = [3, 4, 5, 6; 1]$

$\overline{K} = [6, 7, 8, 9; 1]$ $\overline{C_s} = [2000,\ 3000,\ 4000,\ 5000;\ 1]$

$\overline{C_k} = [40,\ 50,\ 60,\ 70;\ 1]$

From (2), the ranking Index of $\overline{\lambda}$ is

$$R(\overline{\lambda}) = R(2,3,4,5;1)$$

$$= \frac{1}{4}[1\,(2+3+4+5)]$$

= 3.50 = u.

Proceeding similarly, we get other parameters as follows,

$$R(\overline{\mu}) = R(3,4,5,6;1)$$

$$= \frac{1}{4}[1\,(3+4+5+6)]$$

= 4.50 = v.

$$R(\overline{K}) = R(6,7,8,9;1)$$

$$= \frac{1}{4}[1\,(6+7+8+9)]$$

= 7.50 = w.

$$R(\overline{C_s}) = R(2000,\ 3000,\ 4000,\ 5000;\ 1)$$

$$= \frac{1}{4}[1\,(2000+3000+4000+5000)]$$

= 3500 = m.

$$R(\overline{C_h}) = R(40, 50, 60, 70;\ 1)$$

$$= \frac{1}{4}[1\,(40+50+60+70)]$$

= 55 = n.

Using the above values in (1) we get,

The Total Expected Cost is,

$$TEC = m + wn\left[\frac{E(w)^2 + E(w)}{2\{v-u\,E(w)\}}\right]$$

$$= 3500 + (7.50)(55)\left[\frac{(7.50)^2 + 7.50}{2\{4.50-(3.50)\,(7.50)\}}\right]$$

= 2902.

<u>6.3.</u> For Triangular Fuzzy number :

(i) When w = 0.5, that is generalized Fuzzy number will gives moderate value.

Suppose the arrival rate, service rate, batch size service cost rate and holding cost rate are Triangular Fuzzy number represented by,

$\overline{\lambda} = [3, 4, 6; 0.5]$, $\overline{\mu} = [4, 5, 7; 0.5]$

$\overline{K} = [7, 8, 10; 0.5]$ $\overline{C_s} = [\ 3000,\ 4000,\ 6000;\ 0.5]$

$\overline{C_k} = [\ 50,\ 60,\ 80;\ 0.5]$

From (2), the ranking Index of $\overline{\lambda}$ is

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$R(\overline{\lambda}) = R(3,4,6;0.5)$

$= \frac{1}{4}[0.5\ (3+10+6)]$

$= 2.375 = u.$

Similarly, other values are,

$R(\overline{\mu}) = R(4,5,7;0.5)$

$= \frac{1}{4}[0.5\ (4+10+7)]$

$= 2.625 = v.$

$R(\overline{K}) = R(7,8,10;0.5)$

$= \frac{1}{4}[0.5\ (7+16+10)]$

$= 4.125 = w.$

$R(\overline{C_s}) = R(3000,\ 4000,\ 6000;\ 0.5)$

$= \frac{1}{4}[0.5\ (3000+8000+6000)]$

$= 2125 = m.$

$R(\overline{C_h}) = R(50,60,80;\ 0.5)$

$= \frac{1}{4}[0.5\ (50+120+80)]$

$= 31.25 = n.$

Using the above values in (1) we get,

The Total Expected Cost is,

$$TEC = m + wn\left[\frac{E(w)^2 + E(w)}{2\{v - u\,E(w)\}}\right]$$

$$= 2125 + (4.125)(31.25)\left[\frac{(4.125)^2 + 4.125}{2\{2.625 - (2.375)\ (4.1251)\}}\right]$$

$= 1935.$

(i) When w = 1, that is normalized Fuzzy number will gives optimality value 1.

Suppose the arrival rate, service rate, batch size service cost rate and holding cost rate are Triangular Fuzzy number represented by,

$\overline{\lambda} = [3,4,6;1]$, $\quad \overline{\mu} = [4,5,7;1]$

$\overline{K} = [7,8,10;1]$ $\quad \overline{C_s} = [\,3000,\ 4000,\ 6000;\ 1]$

$\overline{C_k} = [\,50,\ 60,\ 80;\ 1]$

From (2), the ranking Index of $\overline{\lambda}$ is

$R(\overline{\lambda}) = R(3,4,6;1)$

$= \frac{1}{4}[1\ (3+8+6)]$

$= 4.25 = u.$

Similarly, other values are,

$R(\overline{\mu}) = R(4,5,7;1)$

$$= \frac{1}{4}[1\,(4+10+7)]$$
$$= 5.25 = v.$$

$$R(\overline{K}) = R(7, 8, 10; 1)$$
$$+ = \frac{1}{4}[1\,(7+16+10)]$$
$$= 8.25 = w.$$

$$R(\overline{C_s}) = R(3000,\ 4000,\ 5000;\ 1)$$
$$= \frac{1}{4}[1\,(3000+8000+5000)]$$
$$= 4000 = m.$$

$$R(\overline{C_h}) = R(50, 60, 80;\ 1)$$
$$= \frac{1}{4}[1\,(50+120+80)]$$
$$= 6250 = n.$$

Using the above values in (1) we get,

The Total Expected Cost is,

$$TEC = m + wn\left[\frac{E(w)^2 + E(w)}{2\{v - u\,E(w)\}}\right]$$
$$= 4000 + (8.25)(62.50)\left[\frac{(8.25)^2 + 8.25}{2\{5.25 - (4.25)(8.25)\}}\right]$$
$$= 33.40.$$

7. Conclusion

In this paper, Fuzzy set Theory has been applied to a Batch Queuing model. Cost measures of Fuzzy Batch Arrival Queuing model have been used in operations and service mechanism for evaluating system performance. Further, the Fuzzy problem has been transformed into crisp problem using Ranking Function method. Since the performance measures such as the total expected cost are crisp values are conclude that the solution of fuzzy problem can be obtained by Ranking Function method very effectively.

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