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STUDY ON PRODUCT ESTIMATION

N.Sahoo* K.B.Panda**
Department of Statistics, Utkal University, Bhubaneswar,India.

Abstract
Exploiting the product estimators due to Srivastava (1983) and Agrawal and Jain (1989), a new product estimator has been proposed. The new product estimator is found to perform better than its competing estimators from the standpoint of bias and mean square error both in one-phase sampling and two-phase sampling under conditions which hold good in practice. The theoretical findings are supported by a numerical illustration.

Key Words: Auxiliary Variable, Product Estimator, Bias And Mean Squared Error, Predictive Estimation.

1. Introduction

Consider a population of size N whose units are arbitrarily labelled $1,2, \ldots\ldots, N$ and let y_i and x_i be the values for the i^{th} unit $(i = 1,2, \ldots. N)$ in respect of the study variable y and the auxiliary variable x, respectively. With a view to estimating the population total $Y = y_1 + y_2 + \cdots + y_N$ or the population mean $\bar{Y} = \frac{Y}{N}$, we consider a sample of size n drawn by simple random sampling without replacement. Under the assumption that y and x are negatively correlated, a possible choice for estimating the population mean $\bar{Y}$ is the customary product estimator given by

$$\bar{y}_p = \frac{\bar{y}\bar{x}}{\bar{X}}, \quad (1.1)$$

Where $\bar{y}$ and $\bar{x}$ are, respectively, the sample means in respect of the study and the auxiliary variables and $\bar{X}$ is the population mean of the auxiliary variable.

Making use of (1.1) as the mean per unit for the unobserved units in the population, Srivastava (1983) invoked the usual predictive approach due to Basu (1971) to suggest the estimator

$$\bar{y}_p' = \frac{n\bar{y}}{N} + \frac{(N-n)^2\bar{y}\bar{x}}{N(N\bar{X}-n\bar{x})}. \quad (1.2)$$

Agrawal and Jain (1989) proposed a predictive product estimator given by

$$\bar{y}_p'' = \frac{\bar{y}\ddot{x}}{\ddot{X}}, \quad (1.3)$$

Where $\ddot{x}$ and $\ddot{X}$ are, respectively, the sample and population harmonic means of x values defined as

$$\ddot{x} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}} \text{ and } \ddot{X} = \frac{N}{\sum_{i=1}^{N}\frac{1}{x_i}}.$$

2. The new Product Estimator

Combining the ideas due to Srivastava (1983) and Agrawal and Jain (1989), we propose the product estimator

$$\bar{y}_p^* = \frac{(N-n)\bar{y}\ddot{x}}{(N\ddot{X}-n\ddot{x})}, \quad (2.1)$$

where the symbols have their usual meanings. The bias and mean square error of the estimator to the 1^{st} degree of approximation, i.e., to $O(n^{-1})$ are given, respectively, by

$$B(\bar{y}_p^*) = \frac{N-n}{Nn}\bar{Y}\left(\frac{N}{N-n}\,\rho C_y C_x + \frac{N^2}{(N-n)^2}C_x^2\right) \quad (2.2)$$

$$MSE(\bar{y}_p^*) = \frac{N-n}{Nn}\bar{Y}^2\left(C_y^2 + \frac{2N}{N-n}\,\rho C_y C_x + \frac{N^2}{(N-n)^2}C_x^2\right). \quad (2.3)$$

Where $C_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\frac{(y_i-\bar{Y})^2}{\bar{Y}^2}$, $C_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\frac{(x_i-\bar{X})^2}{\bar{X}^2}$ and ρ is the correlation coefficient between y and x, assumed to be negative.

3. Comparison of biases

The biases of the competing estimators to $O(n^{-1})$ have been arrived at as follows:

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$$B(\bar{y}_p) = \frac{N-n}{Nn}\bar{Y}\rho C_y C_x \tag{3.1}$$

$$B(\bar{y}_p^{'}) = \frac{N-n}{Nn}\bar{Y}\left(\rho C_y C_x + \frac{n}{N-n}C_x^2\right) \tag{3.2}$$

$$B(\bar{y}_p^{''}) = \frac{N-n}{Nn}\bar{Y}\left(\rho C_y C_x + C_x^2\right) \tag{3.3}$$

$$B(\bar{y}_p^{*}) = \frac{N-n}{Nn}\bar{Y}\left(\frac{N}{N-n}\rho C_y C_x + \frac{N^2}{(N-n)^2}C_x^2\right), \tag{3.4}$$

where the variables y and x assume only positive values. A comparison of absolute values of biases yields the following results:

The estimator $\bar{y}_p^*$ is less biased than $\bar{y}_p$ iff

$$-\frac{N^2}{n(N-n)} < \frac{\rho C_y}{C_x} < -\frac{N^2}{(N-n)(2N-n)}. \tag{3.5}$$

The estimator $\bar{y}_p^*$ is less biased than $\bar{y}_p^{'}$ iff

$$-\frac{N^2-n(N-n)}{n(N-n)} < \frac{\rho C_y}{C_x} < -\frac{N^2+n(N-n)}{(N-n)(2N-n)}. \tag{3.6}$$

and the estimator $\bar{y}_p^*$ is less biased than $\bar{y}_p^{''}$ iff

$$-\frac{2N-n}{N-n} < \frac{\rho C_y}{C_x} < -\frac{2N^2-n(2N-n)}{(N-n)(2N-n)}. \tag{3.7}$$

It may be noted here that the mean per unit estimator and unbiased product type estimator being unbiased estimators do not come under the purview of comparison of biases, see Sukhumi et al. (1980). The conditions (3.5), (3.6) and (3.7) hold good in practice very often.

4. Comparison of Mean Square Errors

The variance or mean square error of mean per unit estimator is given by

$$V(\bar{y}) = MSE(\bar{y}) = \frac{(N-n)}{Nn}\bar{Y}^2 C_y^2. \tag{4.1}$$

The mean square errors of the competing estimators $\bar{y}_p, \bar{y}_{p_1}, \bar{y}_p^{'}, \bar{y}_p^{''}$ are, to O(n^{-1}), found to be same and is as follows:

$$MSE(\bar{y}_p) = MSE(\bar{y}_{p_1}) = MSE(\bar{y}_p^{'}) = MSE(\bar{y}_p^{''}) = \frac{N-n}{Nn}\bar{Y}^2\left(C_y^2 + 2\rho C_y C_x + C_x^2\right). \tag{4.2}$$

Thus, the estimator $\bar{y}_p^*$ is more efficient than the competing estimators $\bar{y}, \bar{y}_p, \bar{y}_{p_1}, \bar{y}_p^{'}, and\ \bar{y}_p^{''}$ if

$$\frac{\rho C_y}{C_x} \le \frac{-(2N-n)}{2(N-n)}. \tag{4.3}$$

Since condition (4.3) implies the usual condition for $\bar{y}_p$ to fare better than $\bar{y}$, the proposed estimator $\bar{y}_p^*$ is also more efficient than $\bar{y}$ under (4.3).

5. Comparison of biases and mean square errors in two- phase sampling

When $\bar{X}$ is not known, we take recourse to two-phase sampling or double sampling. Under the technique, the expressions for the biases and mean square errors of the four estimators to the 1^{st} degree of approximation, i.e, to $O(n^{-1})$ are as given below:

$$B(\bar{y}_{pd}) = \left(\frac{1}{n} - \frac{1}{n'}\right)\bar{Y}\rho C_y C_x \tag{5.1}$$

$$B(\bar{y}_{pd}^{'}) = \left(\frac{1}{n} - \frac{1}{n'}\right)\bar{Y}\left(\rho C_y C_x + \frac{n}{N-n}C_x^2\right) \tag{5.2}$$

$$B(\bar{y}_{pd}^{''}) = \left(\frac{1}{n} - \frac{1}{n'}\right)\bar{Y}\left(\rho C_y C_x + C_x^2\right) \tag{5.3}$$

$$B(\bar{y}_{pd}^{*}) = \left(\frac{1}{n} - \frac{1}{n'}\right)\bar{Y}\left(\frac{N}{N-n}\rho C_y C_x + \frac{N^2}{(N-n)^2}C_x^2\right) \tag{5.4}$$

$$MSE(\bar{y}_{pd}) = MSE(\bar{y}_{p_{1d}}) = MSE(\bar{y}'_{pd}) = MSE(\bar{y}''_{pd}) = \bar{Y}^2\left[\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2 + \left(\frac{1}{n}-\frac{1}{n}\right)\left(2\rho C_y C_x + C_x^2\right)\right] \quad (5.5)$$

and $$MSE(\bar{y}^*_{pd}) = \bar{Y}^2\left[\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2 + \left(\frac{1}{n}-\frac{1}{n}\right)\left\{\frac{2N}{N-n}\rho C_y C_x + \frac{N^2}{(N-n)^2}C_x^2\right\}\right] \quad (5.6)$$

Comparison of biases and mean square errors yield the same results as in the case of one-phase sampling discussed earlier.

6. Numerical illustration

For the purpose of establishing the superiority of $\bar{y}^*_p$ over its competing estimators, we refer to an example from Maddala (1977), wherein a simple random sample of size 4 is drawn from a real population of size 16 with the following population quantities, see Adewara et al (2012):

$$\bar{Y} = 7.6375, \bar{X} = 75.4313, C_y = 0.2278, C_x = 0.0986 \ and \ \rho = -0.6823$$

The biases and mean square errors of the competing estimators have been computed and presented in the following table:

Table 6.1: Bias and MSE of the Competing Estimators

Sl.No.	Estimators	Absolute bias	Mean square error
1	$\bar{y}$	0.0000	0.5675
2	$\bar{y}_{p_1}$	0.0000	0.3390
3	$\bar{y}_p$	0.0219	0.3390
4	$\bar{y}'_p$	0.0173	0.3390
5	$\bar{y}''_p$	0.0080	0.3390
6	$\bar{y}^*_p$	0.0045	0.3095

The above table clearly points to the fact that the proposed estimator performs better than its competing estimators with respect to bias and mean square error. The estimator is found to be almost unbiased and has percentage gain in precision approximately equal to 84 and 10 as compared to the mean per unit estimator and other competing estimators, respectively.

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