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RICCI RECURRENT PROJECTIVE TRANSFORMATION

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Introduction

Let F_n be an n-dimensional Finsler space equipped with fundamental metric function $F(x,\dot{x})$ satisfying the requisite condition for conditions for being a Finsler metric and $g_{ij}(x,\dot{x})$ be the fundamental metric tensor defined by $F^2(x,\dot{x}) = g_{ij}(x,\dot{x})\dot{x}^i\dot{x}^j$, Let $T_j^i(x,\dot{x})$ be any tensor field, the covariant derivative of such a tensor field in the sense of cartain is given by Rund [2].

(1.1) $T^i_{j|k} = \partial_k T^i_j - \dot{\partial}_k T^i_j \partial_k G^h + T^m_j \Gamma^{xi}_{mk} - T^i_m \Gamma^{xm}_{jk}$

Where $\Gamma^{xi}_{jk}(x,\dot{x})$ are the cartan connection coefficients satisfying the relation?

(1.2) (a) $\Gamma^{xi}_{jk} = \Gamma^{xi}_{kj}$, (b) $\dot{\partial}_h \Gamma^{xi}_{jk} \dot{x}^h = 0$.

We are quoting the following commutation formulae which shall be used in the later discussion:

(1.3) $T^i_j|_{hk} - T^i_j|_{kh} = -\dot{\partial}_r T^i_j K^r_{mhk}\dot{x}^m + T^r_j K^i_{rhk} - T^i_r K^r_{jkh}$,

(1.4) $\left(\dot{\partial}_k T^i_j\right)_{|h} - \dot{\partial}_k T^i_j|_k = \dot{\partial}_r T^i_j C^i_{hk|m} x^m - T^r_j \dot{\partial}_k \Gamma^{xi}_{rh} + T^i_r \dot{\partial}_k \Gamma^{xr}_{jh}$.

Where curvature tensor field $K^i_{jkh}(x,\dot{x})$ is given by

$$(1.5) K^r_{jkh} = Q_{(h,k)}\left\{\frac{\partial \Gamma^{xi}_{jh}}{\partial x^k} - \frac{\partial \Gamma^{xi}_{jh}}{\dot{\partial} x^r} G^r_k + \Gamma^{xi}_{mk}\Gamma^{xm}_{jh}\right\}$$

Where $Q_{(h,k)}\{\dots\dots\dots\}$ stands for the interchange of the indices h and k and subtraction thereafter. The curvature tensor field $K^i_{jkh}(x,\dot{x})$ is a homogenous function of degree zero in its directional arguments. The term $C^i_{jk}(x,\dot{x})$ appearing in (1.4) is any tensor field give by the following equation:

$$(1.6) C_{ijk}(x,\dot{x}) = \frac{1}{2}\frac{\partial g_{ij}(x,\dot{x})}{\partial \dot{x}^k} = \frac{1}{4}\frac{\partial^3 F^2(x,\dot{x})}{\partial \dot{x}^i\,\partial \dot{x}^j \partial \dot{x}^k}$$

We now give the following definition which shall be used in the later discussions.

If the recurrent normal projective curvature tensor of a Finsler space be assumed to be decomposable in the form (1.6) then such a space is always normal projective recurrent.

We now consider the last case in which the normal projective curvature tensor of the Finsler space is decomposable in the form (1.5). Transvecting (1.6) by x^j and thereafter using (1.4), we get

(2.1) $H^i_{kh} = X^i_k x^j Y_{jh}$

Transvecting (2.1) by y_i, we get

(2.2) $X^i_j x^j \; y_i Y_{kh} = 0$,

Form (2.2) we conclude that either of the following two conditions will always hold the space under consideration
(2.3) $(a) X_k^i x^j y_i = 0, \quad (b) Y_{kh} = 0$

If the condition (2.3b) automatically leads to $N_{jkh}^i = 0$, therefore such a condition is always not possible. Hence, only alternative left with us is to consider the case:

(2.4) $X_j^i x^j y_i = 0$

Contracting (2.1) with respect to the indices i and j we get

(2.5) $H_{rkh}^r = H_{hk} - H_{kh} = X_r^r Y_{kh}$

From (2.5), we have

(2.6) $Y_{kh} = \frac{1}{X}(H_{hk} - H_{kh}) \quad where \quad X = X_r^r$

Therefore, we can state:

If the normal projective curvature tensor of a Finsler space be assumed to be decomposable in the form (2.7) then the tensors $X_j^i$and Y_{kh}always satisfy (2.4) (2.5)and (2.6).

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