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BAYESIAN ESTIMATION OF EXPONENTIAL DISTRIBUTION USING TYPE I CENSORED DATA UNDER SQUARED ERROR LOSS FUNCTION

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Abstract
In life testing experiment, it so often happens that the experiment is censored in the sense that the experimenter may not be in a position to observe the life times of all items put on test because of time limitations and other restrictions on the data collection. In this paper, Bayesian approach for exponential distribution based on type-I censoring scheme is considered. Bayes point estimates under squared error loss and credible intervals for the exponential distribution based on type-I censoring scheme is discussed.

Key Words: Prior distribution, Posterior distribution, censoring, loss function.

1. Introduction

Balakrishnan,(1990), proposed exponential distribution plays an important role in survival and engineering reliability. Bayesian estimation of parameters based on Type I and Type II censored samples have been discussed by several authors in the literature. Sarhan (2003) studied empirical Bayes estimates in one parameter exponential distribution. Zhou(1998) considered Bayes estimation and prediction for one and two parameter exponential distribution. Singh et al (2007) proposed Bayes point estimates for the scale parameter under type II censoring by using generalized non-informative prior and natural conjugate prior. Limited work has been done under single parameter of type I censored data. The main objective of this paper to obtain Bayes point estimates and credible intervals for the scale parameter of exponential distribution based on Type I censored sample. In type-I censored samples, the experiment continue upto a preselected fixed time T but number of failures is random.

2. Bayesian Estimation

2.1. Model

Let $x_1, x_2, \ldots x_n$ be a random sample of size 'n' drawn from exponential distribution with parameter ¯ then p.d.f of x is given by

$$f(x/¯) = ¯ e^{-¯ x} \qquad (1)$$

$$F(x) = 1 - e^{-¯ x}; x \geq 0 \qquad (2)$$

2.2. Type I censored sample

If a random sample of 'n' units is tested until a predetermined time T at which time the test is terminated. Failure time for 'r' observations are observed, where 'r' is a random variable. Thus the lifetimes are observed only if

$$¢_i = \begin{cases} 0, & x_i > T \\ 1, & x_i \leq T \end{cases} \qquad (3)$$

Therefore $k = \sum ¢_i$, where k>0

Likelihood function

The likelihood function in this case is given by

$$L(x/¯, T) = \prod_{i=1}^{n} [f(x_i, ¯)]^{¢_i} [f(T/¯)]^{1-¢_i}$$

$$= \prod_{i=1}^{n} (¯ e^{-¯ x_i})^{¢_i} (1 - (1 - e^{-¯ T})^{1-¢_i}$$

$$= ¯^{\sum ¢_i} e^{-¯ \sum_i x_i ¢_i} e^{-¯ T(\sum (1-¢_i))}$$

$$= ¯^{k} e^{-¯ \sum_i x_i ¢_i} e^{-¯ T(n-k)}$$

$$= ¯^{k} e^{-¯ [\sum_i x_i ¢_i + T(n-k)]}$$

2.3. Prior Distribution

The parameter ¯ is assumed to follow gamma distribution with p.d.f. given by:

$$®(¯) = \frac{\sim}{\Gamma\sim} ¯^{\sim-1} e^{- ¯} ; \quad \sim, \quad > 0 \qquad (5)$$

2.4. Posterior Distribution Of ‘ ¯ ’

The posterior distribution of ¯ given ($x_1, x_2, \ldots x_n ; T$) is

$$p(¯ / x, T) = \frac{L(x/¯, T)®(¯)}{\int_0^\infty L(x/¯, T)®(¯) d¯}$$

$$= \frac{\Gamma\sim ¯^{k} e^{-¯[\sum_i x_i¢_i + T(n-k)]} \sim ¯^{\sim-1} e^{- ¯}}{\sim\Gamma\sim \int_0^\infty ¯^{k} e^{-¯[\sum_i x_i¢_i + T(n-k)]} ¯^{\sim-1} e^{- ¯} d¯} \qquad = \frac{¯^{(k+\sim)-1} e^{-¯[\sum_i x_i¢_i + T(n-k) + \ ]}}{\int_0^\infty ¯^{(k+\sim)-1} e^{-¯[\sum_i x_i¢_i + T(n-k) + \ ]} d¯}$$

$$= \frac{[\sum_i x_i¢_i + T(n-k) + \ ]^{k+\sim} ¯^{(k+\sim)} e^{-¯[\sum_i x_i¢_i + T(n-k) + \ ]}}{\Gamma k + \sim} \qquad (6)$$

2.5: Bayes Estimate Of ‘ ¯ ’

Bayes point estimate of $\hat{¯}$ of ¯ under squared error loss is the mean of posterior p.d.f. of ¯,which is given by

$$\hat{¯}_1 = E_p[¯] = \int_0^\infty ¯ p(¯ / x) d¯$$

$$= \frac{[\sum_i x_i¢_i + T(n-k) + \ ]^{k+\sim}}{\Gamma k + \sim} \int_0^\infty ¯^{(k+\sim+1)-1} e^{-¯[\sum_i x_i¢_i + T(n-k) + \ ]} d¯$$

$$= \frac{[\sum_i x_i¢_i + T(n-k) + \ ]^{k+\sim} \Gamma k + \sim + 1}{\Gamma k + \sim [\sum_i x_i¢_i + T(n-k) + \ ]^{k+\sim+1}}$$

$$= \frac{k + \sim}{\sum_i x_i¢_i + T(n-k) + } \qquad (7)$$

4.3. Simulation study:

In this section, the performance of Bayes estimator ‘ ¯ ’ is investigated through a simulation study based on various Type-I censored schemes. The simulation study is carried out for various of the combination (¯ n,t): for ¯ =0.5, ~ =2.5, =3, the termination T is assumed arbitrary to equal 1.9 and 2.2 with n=5,10,15,20,30,50,75,100. N=1000 simulated data sets are generated from exp (¯) by using R-software. For purpose of comparison, the proposed Bayes estimates ie BSE along with mean square error (MSE) are presented in table1 and 2. Estimators with small MSE values are preferred.

Table:1 Bayes estimators along with MSE when ¯ =0.5, ~ =2.5, =3,T=1.9 based Type-I censored sample

n	Criteria	BSE	n	Criteria	BSE
5	Estimated value	0.2205	30	Estimated value	0.6370
	MSE	0.0781		MSE	0.0188
10	Estimated value	0.2736	50	Estimated value	0.8124
	MSE	0.0512		MSE	0.0976
15	Estimated value	0.3257	75	Estimated value	1.4616
	MSE	0.0304		MSE	0.9247
20	Estimated value	0.3236	100	Estimated value	1.7223
	MSE	0.0311		MSE	1.4941

From the above table: 1.It is seen as the sample size increases MSE decreases.

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Table: 2 Bayes estimators along with MSE when ¯ =0.5, ~ =2.5, =3,T=2.2 based Type-I censored sample

n	Criteria	BSE	n	Criteria	BSE
5	Estimated value	0.2876	30	Estimated value	1.6078
	MSE	0.0451		MSE	1.2273
10	Estimated value	0.5438	50	Estimated value	3.2162
	MSE	0.0019		MSE	7.3782
15	Estimated value	0.8837	75	Estimated value	13.2004
	MSE	0.1472		MSE	161.2992
20	Estimated value	0.9814	100	Estimated value	16.8977
	MSE	0.2318		MSE	368.8856

From the above table.2.It is seen as the sample size increases MSE decreases.

4. Conclusions

It is shown from simulation study that MSE decreases with increases in sample size. Also it is found that as censor time increases MSE decreases.

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